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DETERMINING MAXIMUM VALUES OF GRAVITATIONAL DRIFT AND DRIFT FROM UNEQUAL RIGIDITY OF FLOATING INTEGRATION GYROSCOPES

G. A. Slomyanskiy

The moment of interferences, affecting the gyro unit of a floating integration gyroscope, consists in the general case of moments, not depending upon acceleration and moments, depending upon acceleration. To the first one blongs moments, created by flexible current conductors and convectional liquid streams, reactive moments of angle and moment sensing elements, etc. The moments depending upon acceleration, appear to be moments due to unequal suspension and unequal rigidity of the gyro unit. The origination of treatments of the gyroscope because of unequal rigidity of the system of suspension was first mentioned by A. Yu. Ishlinskiy [1].

During the movement of the instrument with linear acceleration $\bar{a}$, the lift and weight (more accurately - "apparent weight") of the gyro unit will be accordingly equal [2] $m_1 \omega$ and $m\omega$ ($m_1 = \rho V$-mass of the liquid in volume V of the gyro unit, m - mass of gyro unit, $\omega = |\bar{g} - \bar{a}|$, g - acceleration of gravitational force, and the floating axis of the gyro unit will be

directed by the "apparent vertical". Consequently at $a \neq 0$, the difference between the lifting force and weight force of the gyro unit changes in $\bar{g} - \bar{a}/g$ once in comparison with its value at $a = 0$, the differential sign here remains unchanged. As a result of this, the moment, due to unequal suspension of the gyro unit, will be proportional to the first degree of vector projection $\bar{w} = \bar{g} - \bar{a}$ on the lateral plane of the instrument yz_0 (y and z_0 output and transverse axes of the instrument). Accordingly also the drift, caused by this moment, will also be proportional to the first degree of vector projection w on plane yz_0.

Since the drift is due to unequal suspension of the gyro unit, it appears to be an acceleration function, then it should be evaluated in $\frac{\text{degr/hr}}{g}$, i. e., to evaluate by the value of angular rate of drift, observed on the ground in the absence of transferable accelerations ($a = 0$). The drift due nonequal suspension of the gyro unit, observed on the ground in the absence of transfer accelerations ($a = 0$) and during horizontal position of the output axis of the instrument x, will be called gravitational drift. At $a \neq 0$ and horizontal position of the axis x the drift, due to unequal suspension of the gyro unit, will be equal to gravitational drift, multiplied by the expressed in fractions, g value of vector projection $\bar{w} = \bar{g} - \bar{a}$ on plane yz_0.

In floating gyroscopes the moment due to unequal regidity of the gyro unit, originates because, that in an unequal regidity gyro unit the elastic displacements of the center of gravity and the center of pressure take place in directions, not coinciding with the direction of action of the forces, causing these displacements. This moment, and consequently, the drift caused by it, are proportional to the second degree of vector projection $\bar{w}' = \bar{g} - \bar{a}$ on the

transverse plane of the instrument yz_0. In conformity with this, the drift due to unequal rigidity of the gyro unit, should be evaluated in $\frac{\text{degr/hr}}{g^2}$, i. e., evaluated by the value of angular speed of the drift, observed on the ground in the absence of transferable accelerations ($a = 0$).

At first, the gyro unit of floating integrating gyroscopes consisted of a frame with gyromotor and hollow cylinder, which was fitted over the si side sections of the frame, made in the form of disks, and hermetically connected with it. But with the same time of adding to the gyro unit of greater rigidity, the frame was removed and its functions were imposed directly on the float. The float consisted of two parts (each one in the form of a glass) hermetically connected with each other. At first, in one of them the gyromotor was secured. At such gyro unit construction, efforts should be made that the gyro motor should be equi-rigid, and the float - practically absolutely rigid with respect to maximum possible values of the forces affecting same.

Assuming, that the float appears to be practically absolutely rigid, and the construction of the gyro motor does not satisfy the condition of equi-rigidity and it is made with a sationary axis, we will bring out expressions for moments M_2 and M_3, due congruently to unequi-suspension and unequi-rigidity of the gyro unit. For this we will introduce the following designations (see figure).

Point O-trace of output axis of the instrument x (axes of rotation of gyro unit), directed behind the plane of the drawing;

z - axis of rotation of gyro motor rotor;

y - axis, perpendicular to axes x and z;

η and ξ - horizontal and vertical axes;

θ - angle of inclination of axis z to the plane of the horizon;

g - acceleration of gravitation force;

m_1- mass of liquid in volume of gyro unit;

r_1; ψ_1 - polar coordinates of center of pressure O_1;

m_2 - mass of gyro unit without gyro motor;

r_2; ψ_2 - polar coordinates of point O_2 of application of weight force m_2g;

m_3 - mass of gyro motor;

r_3; ψ_3 - polar coordinates of point O_3 in which the center of mass of the gyro motor would be situated, if the gyromotor would be absolutely rigid;

m_4 - mass of gyromotor rotor unit, subjected to elastic displacement parallel to axis y as well as parallel to axis z.

m_3-m_4 - mass of axis unit of gyromotor with stator, subjected to elastic displacement only parallel to axis y;

O_4 - point of application of weight force $(m_3-m_4)g$;

O_5 - point of application of weight force m_4g;

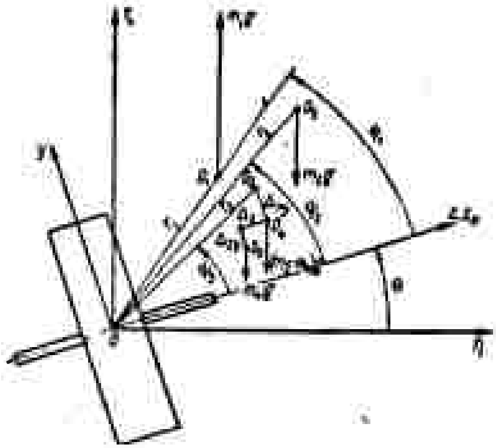

Figure. For the derivation of expression for moment $M_{2,3}$ due to unequal suspension and unequal rigidity of the unit (gyro unit).

$m = m_2 + m_3$ - weight mass of gyro unit;

r, ψ - polar coordinates of the center of mass of the entire gyro unit at absolutely rigid gyromotor;

K_{3u} - transverse rigidity of the unit of the axis with stator;

K_{4u} and K_{4z} - correspondingly radial and axial rigidity of the unit of rotor with bearings;

$$\Delta_{1y} = \frac{gm_3}{K_{3y}} \cos\theta \qquad (1)$$

- elastic displacement of center of unit mass of the axis with stator parallel to axis y;

$$\Delta_{2y} = \frac{gm_4}{K_{4y}} \cos\theta; \quad \Delta_z = \frac{gm_4}{K_{4z}} \sin\theta \qquad (2)$$

- elastic displacement of center of rotor unit mass with bearings relative to the center of mass of the axis unit and stator parallel to y and z respectively.

On the basis of the expressions (1) and (2) we obtain, that the forces, depicted in the figure create around axis x, a moment

$$M_{2.3} = gmr\cos(\theta+\psi) - gm_1r_1\cos(\theta+\psi_1) + \\ + \frac{g^2}{2}\left[\frac{m_3^2}{K_{3y}} + m_4^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right)\right]\sin 2\theta, \quad (3)$$

where $$r = \frac{1}{m}\sqrt{(m_2r_2)^2 + (m_3r_3)^2 + 2m_2m_3r_2r_3\cos(\psi_2 - \psi_3)},$$

$$\operatorname{tg}\psi = \frac{m_2r_2\sin\psi_2 + m_3r_3\sin\psi_3}{m_2r_2\cos\psi_2 + m_3r_3\cos\psi_3}.$$

In expression (3) the first two components represent a moment M_2, due to uniequi-suspension of the gyro unit relative to axis x. In this way

$$M_2 = g[mr\cos(\theta+\psi) - m_1r_1\cos(\theta+\psi_1)]. \quad (4)$$

The third component appears to be the moment M_3, due to uniequi-rigidity of gyromotor. Consequently,

$$M_3 = \frac{g^2}{2}\left[\frac{m_3^2}{K_{3y}} + m_4^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right)\right]\sin 2\theta. \quad (5)$$

Maximum value of moment M_2:

$$M_{2\,max} = g\sqrt{(mr)^2 + (m_1r_1)^2 - 2mm_1rr_1\cos(\psi_1 - \psi)} \quad (6)$$

is obtained at an angle θ^*, equaling

$$\theta^* = \operatorname{arc\,tg}\frac{m_1r_1\sin\psi_1 - mr\sin\psi}{mr\cos\psi - m_1r_1\cos\psi_1}. \quad (7)$$

Using equalities (6) and (7) we can represent equation (4) in the form of

$$M_2 = M_{2\,max}\cos(\theta - \theta^*). \quad (8)$$

Moment M_2 principally can be made equal to zero for any given values of angle θ by various methods. But in order, that $M_2 = 0$ at any given possible values of mass m_1, which appears to be the the function of temperature of the liquid, it is necessary to attain the thing, that $r = r_1 = 0$, i. e., that the center of mass and the center of pressure of the gyro unit should

lie on its axis of rotation. At any other given method of turning into the moment M_2 equality $M_2 = 0$ will not be invariant with respect to possible changes in mass m_1. And so, for example, if at a certain liquid temperature moment M_2 is turned into zero by bringing the center of mass and center of pressure into one point, not lying on the axis of rotation of the gyro unit ($m = m_1, \psi = \psi_1$, $r = r_1 \neq 0$) then at a change in temperature the equality $m_1 = m$ is disrupted because of change in m_1, in consequence of which $r = r_1 \neq 0$, moment M_2 will cease being zero.

To attain this, that the moment M_2 should be accurately and stably equal, zero is very difficult. A reduction of moment M_2 to a value close to zero, is obtained by thorough balancing of the gyro unit by a special technology with the use of highly precision devices.

From equation (5), it is evident that at a change in angle θ, moment M_3 changes with dual frequency, and its amplitude value

$$M_{3\,max} = \frac{g^2}{2}\left[\frac{m_3^2}{K_{3y}} + m_4^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right)\right] \qquad (9)$$

is obtained at angles

$$\theta = \frac{\pi}{4}(2k+1);\ k = 0, 1, 2, \ldots \qquad (10)$$

In order that moment M_3 should be equal to zero at any given value of angle θ, it is necessary to observe the following condition

$$\frac{m_3^2}{K_{3y}} + m_4^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right) = 0,$$

which also appears to be equirigidity condition in the case under question. When an accurate fulfillment of this condition appears to be difficult, the remaining value of the moment, due to unequi-rigidity, may be compensated with the aid of a special compensator, representing a small mass, fastened

on a flat spring. Turning the spring around its longitudinal axis, it is also possible to change the direction of its deformation, and consequently, also the direction of mass displacement. Selecting experimentally the position of the spring, it is possible to obtain principally total equirigidity of the system. The given compensator reduces the rigidity of the gyro unit in the direction of that axis, along which it has the greater rigidity.

In the general case, the drift of a floating integration gyroscope is caused by the moment

$$M = M_1 + M_2 + M_3,$$

which is active around axis x, where M_1 - moment of interferences, not depending upon acceleration, in other words, a moment due to causes, not connected with the unequi suspension and unequirigidity of the gyro unit. Moments M_2 and M_3 are determined by formulas (8) and (5). Correspondingly, in the general case the angular velocity of the drift

$$\omega_{\text{д}} = \omega_{\text{д}1} + \omega_{\text{д}2} + \omega_{\text{д}3}, \quad (11)$$

where

$$\omega_{\text{д}1} = \frac{M_1}{H}, \quad \omega_{\text{д}2} = \frac{M_2}{H}, \quad \omega_{\text{д}3} = \frac{M_3}{H}. \quad (12)$$

Here H - natural (kinetic) moment of gyroscope.

We will designate

$$\omega_{\text{д}1\,\max} = \frac{M_{1\,\max}}{H} \quad (13)$$

- maximum value of drift velocity, due to the moment of interferences, not depending upon acceleration:

$$\omega_{\text{д}2\,\max} = \frac{M_{2\,\max}}{H} \quad (14)$$

- maximum value of drift velocity, due to unequisuspension of gyro unit;

$$\omega_{д3\,max} = \frac{M_{3\,max}}{H} \quad (15)$$

- maximum value of drift velocity, due to unequirigidity of the gyro unit (in our case, gyromotor).

When planning a floating integration tyroscope, the kinetic moment J and quality D of the gyromotor ($D = H/P_3$, P_3 - weight of gyro motor) should be selected in dependence upon the given values de2max and d3max by the following formulas, obtained from expressions (14), (6) and (15), (9);

$$\frac{ar}{D}(1 + bc) = \omega_{д2\,max}; \quad (16)$$

$$\frac{H}{D^2}\left[\frac{1}{K_{3y}} + d^2\left(\frac{1}{K_{4y}} - \frac{1}{K_{4z}}\right)\right] = \omega_{д3\,max}, \quad (17)$$

where $a = P/P_3$ (P - weight of gyro unit); $b = Q/P)Q$ - weight of liquid in volume of gyro unit); $c = r_1/r$; $d = P_4/P_3$ (P_4 - weight of gyromotor-rotor). Remaining designation as before.

Formula (16) was obtained in the assumption, that $\psi_1 - \psi = \pi$, because in this case moment M_{2max}, determinable by formula (6), has at other equal conditions the maximum value.

In order that the component of angular drift velocity, which does not depend upon acceleration, should not exceed the maximum permissible value dlmax, it is necessary to observe the following inequality

$$M_{1\,max} < H\omega_{д1\,max}. \quad (18)$$

At an equirigid gyromotor H and D are recommended to be determined by formulas (18) and (16).

As is known, the angular velocity of the drift of floating integrating gyroscopes is determined with the aid of a special dynamic stand, which allows to measure same, at horisontal position of axis x and various values of angle

0, as well as in the case, when axis x is vertical. We will assume that the angular velocity of the drift, measured at angles 0 equal to 0, $\frac{}{4}$ and $\frac{}{2}$, equals correspondingly $\omega_D(0)$, $\omega_d\left(\frac{\pi}{4}\right)$ and $\omega_d\left(\frac{\pi}{2}\right)$. The angular velocity of the drift, measured at vertical position of axis x, will be designated by $\omega_{d.vert}$. This velocity can be adopted as velocity ω_{d1}, because at vertical position of axis x, $M_2 = M_3 = 0$ and, consequently, the drift is caused only by moment M_1.

We will show how, knowing $\omega_{d.vert.}$; $\omega_d(0)$; $\omega_d\left(\frac{\pi}{4}\right)$ and $\omega_d\left(\frac{\pi}{2}\right)$, it is possible to determine ω_{d2max} and ω_{d3max}.

From equation (4) and (6) it is evident, that

$$M_{2\,max} = \sqrt{M_2^2(0) + M_2^2\left(\frac{\pi}{2}\right)}, \tag{19}$$

where $M_2(0)$ and $M_2\left(\frac{\pi}{2}\right)$ - values of moment M_2 respectively at $\theta = 0$ and $\theta = \frac{\pi}{2}$.

Having replaced expression (19) in (14), we will obtain

$$\omega_{д2\,max} = \sqrt{\omega_{д2}^2(0) + \omega_{д2}^2\left(\frac{\pi}{2}\right)}, \tag{20}$$

where $\omega_{d2}(0)$ and $\omega_{d2}\left(\frac{\pi}{2}\right)$ - angular velocities of gravitational drift respectively at $\theta = 0$ and $\theta = \left(\frac{\pi}{2}\right)$. From expression (5) and formula (12) for ω_{d3}, it is evident that at these values of angle θ, the angular velocity of the drift $\omega_{d3} = 0$ and, consequently, by formula (11)

$$\left.\begin{aligned} \omega_{д2}(0) &= \omega_д(0) - \omega_{д.\,верт}, \\ \omega_{д2}\left(\frac{\pi}{2}\right) &= \omega_д\left(\frac{\pi}{2}\right) - \omega_{д.\,верт}. \end{aligned}\right\} \tag{21}$$

Substituting (21) in (20), we obtain a final formula for the calculation of maximum value of gravitational drift:

$$\omega_{Д2\,max} = \sqrt{\left[\omega_Д(0) - \omega_{Д.\,верт}\right]^2 + \left[\omega_Д\left(\frac{\pi}{2}\right) - \omega_{Д.\,верт}\right]^2}. \qquad (22)$$

From (8) it is evident, that ω_{d2max} will be observed at angles θ, equal θ^* and $\pi + \theta^*$. At both angles the direction of the drift will be identical.

$$\theta^* = \operatorname{arc\,tg} \frac{M_2\left(\frac{\pi}{2}\right)}{M_2(0)}.$$

Using equality (12) for ω_{d2} and formula (21), we will obtain

$$\theta^* = \operatorname{arc\,tg} \frac{\omega_{Д2}\left(\frac{\pi}{2}\right) - \omega_{Д.\,верт}\ vert}{\omega_{Д2}(0) - \omega_{Д.\,верт}\ vert}. \qquad (23)$$

From expressions (5), (9) and (15), it is directly evident that $_{d3max}$ will take place at angle θ, determinable by equality (10) and, particularly at $\theta = \frac{\pi}{4}$. Consequently, on the basis of equation (11) we can write

$$\omega_{Д3\,max} = \omega_Д\left(\frac{\pi}{4}\right) - \omega_{Д2}\left(\frac{\pi}{4}\right) - \omega_{Д.\,верт}\ vert, \qquad (24)$$

where $\omega_{d2}\frac{(\pi)}{(4)}$ - the value ω_{d2} at $\theta = \frac{\pi}{4}$.

Having adopted in expression (4) $\theta = \frac{\pi}{4}$, we will obtain

$$M_2\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\left[M_2(0) + M_2\left(\frac{\pi}{2}\right)\right].$$

Substituting this value $M_2\left(\frac{\pi}{4}\right)$ in equation (12), for ω_{d2} we obtain, that

$$\omega_{Д2}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\left[\omega_{Д2}(0) + \omega_{Д2}\left(\frac{\pi}{2}\right)\right].$$

Having substituted the given expression and equation (21) in (24) we obtain finally a formula for the calculation of maximum drift value, due to unequirigidity of the gyro unit:

$$\omega_{д3\,max} = \omega_д\left(\frac{\pi}{4}\right) - \frac{\sqrt{2}}{2}\left[\omega_д(0) + \omega_д\left(\frac{\pi}{2}\right)\right] + (\sqrt{2} - 1)\,\omega_{д.\,верт} \cdot vert \qquad (25)$$

In this way, we calculate values ω_{d2max}, ω_{d3max} and θ^* it is necessary to know, as is evident from formulas (22), (25) and (23), the angular velocities of the drift $\omega_d(0)$, $\omega_d\left(\frac{\pi}{4}\right)$, $\omega_d\left(\frac{\pi}{2}\right)$ and $\omega_{d,vert.}$. It should however, be mentioned that at horizontal position of axis x, the rate ω_{d1} may differ somewhat from $\omega_{d.vert}$. Furthermore, together with the mentioned simple method of determining ω_{d2max} and ω_{d3max}, another method should also be recommended, which consists in measuring ω_d at a horizontal position of the axis x and various values of angle θ from 0 to 360°, within equal intervals, e. g., within 30°. Then process the dependence graph ω_d from θ by the harmonic balance method.

Literature

1. A. Yu. Ushlinskiy: Mechanics of special gyroscopic systems. Acad. of Sc. Ukr. SSR, 1952.

2. Slomyanskiy, G. A.: Buoyancy of bodies, submerged in a liquid, situated at relative peace. Izv. Akademii Nauk USSR, Otdel. Tekhn. Nauk, 1958, No. 2.